# Section 4.3
# Equipping Your Function Toolkit

| Curriculum Outcomes | Related Activities | Page in Text |
|---|---|---|
| • model real-world phenomena with linear, quadratic, exponential and power equations, and linear inequalities | • four connected investigations that will explore parameter changes in the graphs of functions<br>• an introduction to the absolute value function is given for those students not familiar with the absolute value function | 174,176, 178,180<br>183 |
| • analyze and describe transformations of quadratic functions and apply them to absolute value functions<br>• express transformations algebraically and with mapping rules<br>• graph equations and inequalities and analyze graphs both with and without graphing technology<br>• apply transformations when solving problems<br>• use transformations to draw graphs | | |

Feb 5-8:16 PM

# Transformations

Transformations are a form of math where we are comparing two different graphs to see how they have moved. There are 4 different types of transformations that we will be talking about in this unit.

In order to compare graphs, we must first graph our equations.

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**Create a table of values (using the same values as seen below) and graph each of the following.**

1. $y = x^2$

2. $y = -x^2$ ⟶ $-(-3)^2 = -9$

3. $y = x^2 + 3$

4. $y = x^2 - 6$

5. $y = 2x^2$

$y = x^2$

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |

(-3, 9)
(-2, 4)

How to find the "y" in the table of values:

Example:

Using $y = x^2$

$y = x^2$
$y = (-3)^2$
$y = 9$

Your first point to plot will be (-3, 9)

**Compare each graph to the $y=x^2$ graph**

- $y = -x^2$ ---> what changed?
- $y = x^2 + 3$ ---> what changed?
- $y = x^2 - 6$ ---> what changed?

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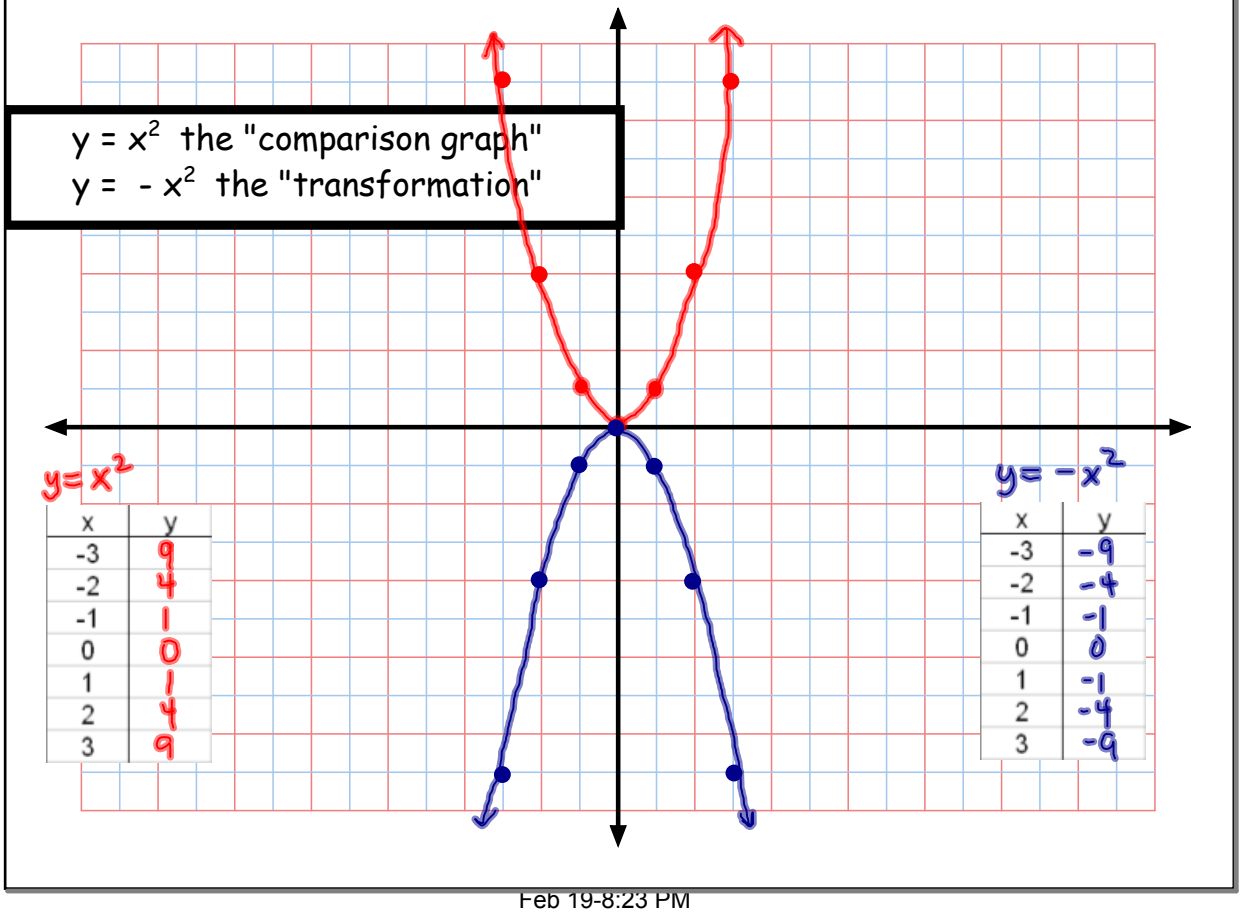
$y = x^2$ the "comparison graph"
$y = -x^2$ the "transformation"



$y = x^2$

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

$y = -x^2$

| x | y |
|---|---|
| -3 | -9 |
| -2 | -4 |
| -1 | -1 |
| 0 | 0 |
| 1 | -1 |
| 2 | -4 |
| 3 | -9 |

Feb 19-8:23 PM

$y = -x^2$

## *How do the 2 graphs compare?*

**We can describe how the 2 graphs compare verbally....**

"Reflected" or "Flipped" over the x-axis.

**And we can describe how the 2 graphs compare mathematically, using what we call "mapping notation"**

**$y = x^2$**

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

**$y = -x^2$**

| x | y |
|---|---|
| -3 | -9 |
| -2 | -4 |
| -1 | -1 |
| 0 | 0 |
| 1 | -1 |
| 2 | -4 |
| 3 | -9 |

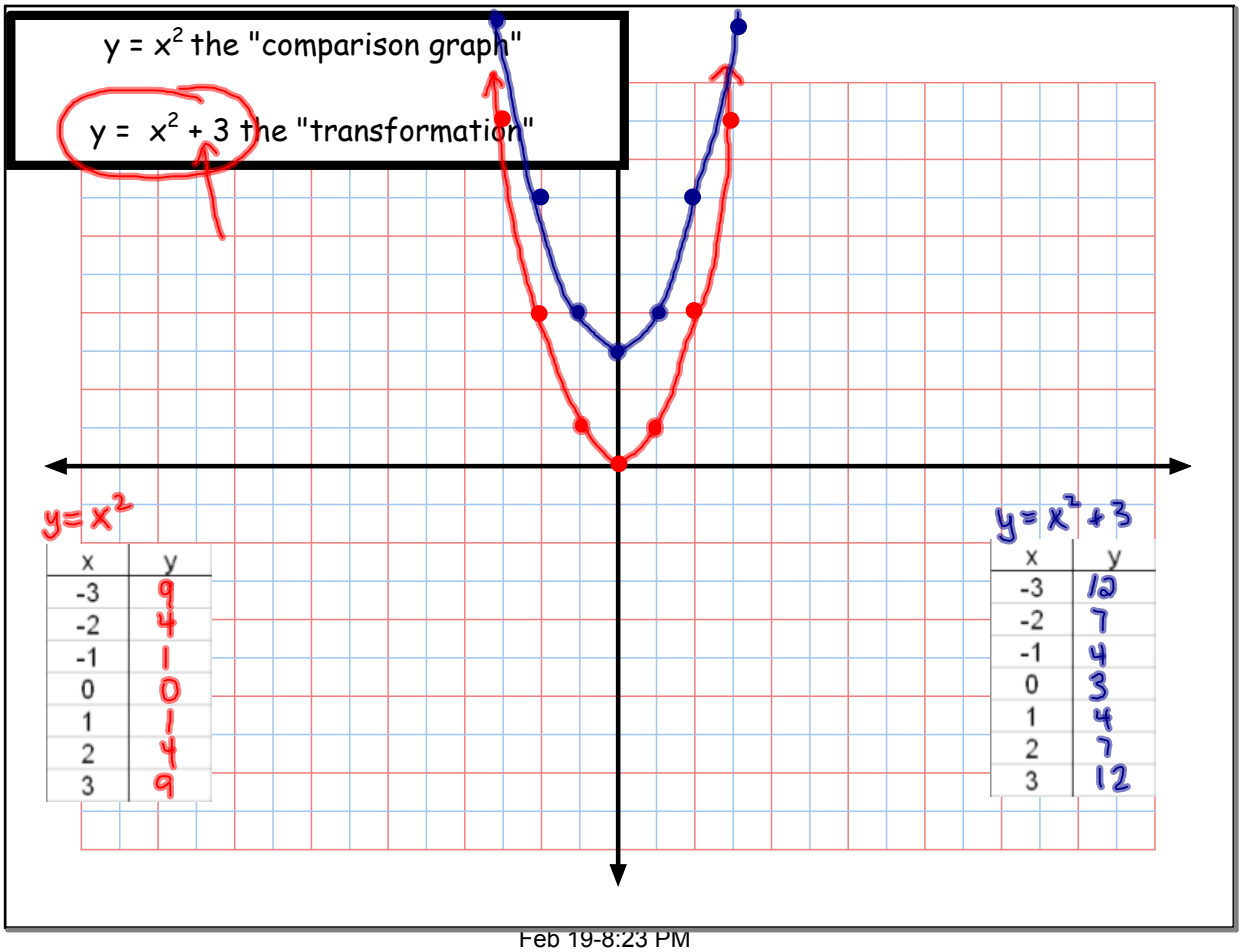
Notice: Each y-value was multiplied by a negative.

$(x, y) \longrightarrow (x, -y)$

Jan 27-11:13 PM

$y = x^2$ the "comparison graph"

$y = x^2 + 3$ the "transformation"



$y = x^2$

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

$y = x^2 + 3$

| x | y |
|---|---|
| -3 | 12 |
| -2 | 7 |
| -1 | 4 |
| 0 | 3 |
| 1 | 4 |
| 2 | 7 |
| 3 | 12 |

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$y = x^2 + 3$

## *How do the 2 graphs compare?*

**We can describe how the 2 graphs compare verbally....**

→ "Translation" or "slide" up 3 units

**And we can describe how the 2 graphs compare mathematically, using what we call "mapping notation"**

**$y = x^2$**

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

**$y = x^2 + 3$**

| x | y |
|---|---|
| -3 | 12 |
| -2 | 7 |
| -1 | 4 |
| 0 | 3 |
| 1 | 4 |
| 2 | 7 |
| 3 | 12 |

Notice:
Each y-value had 3 added to it.

$(x, y) \longrightarrow (x, y+3)$

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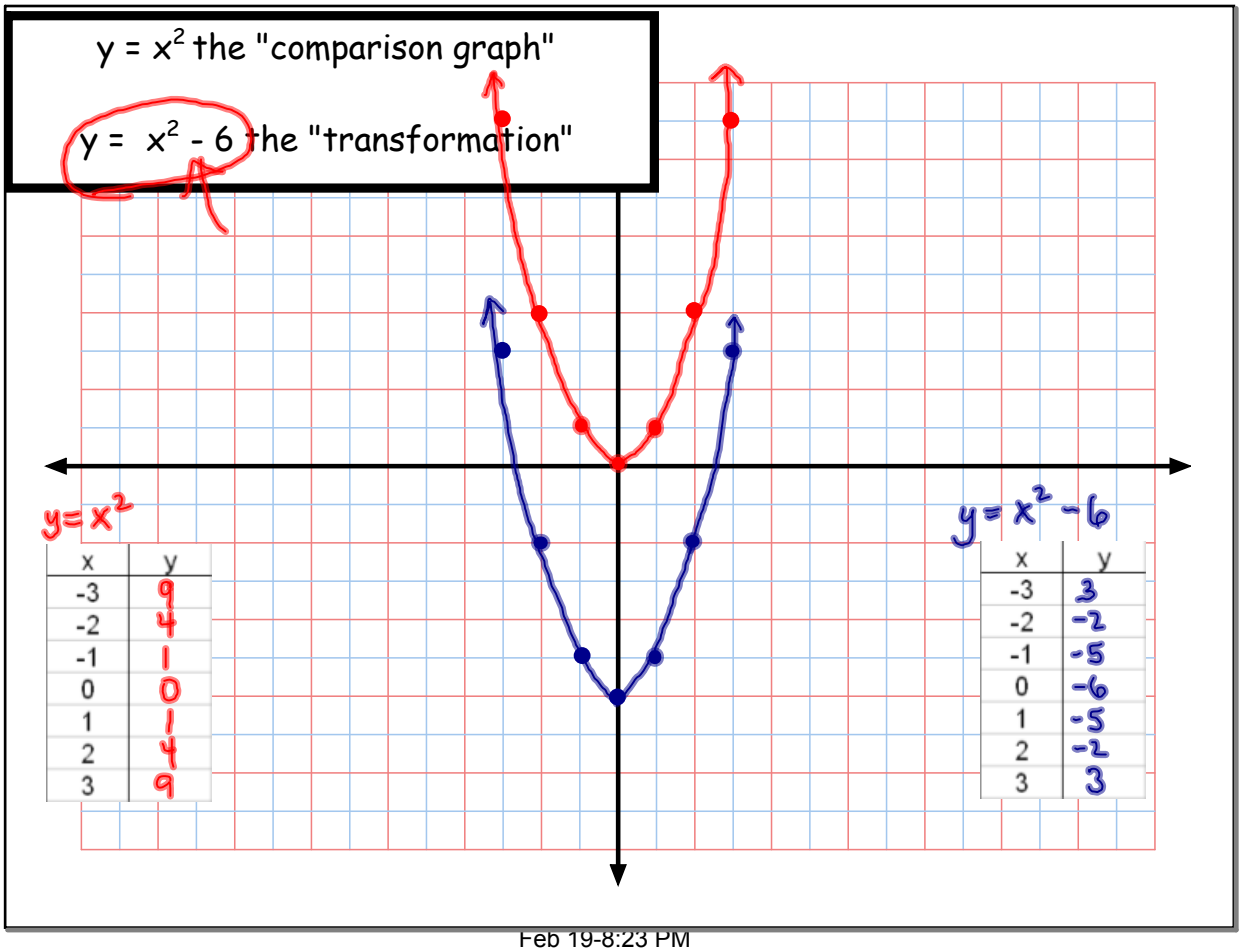
$y = x^2$ the "comparison graph"

$y = x^2 - 6$ the "transformation"



$y = x^2$

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

$y = x^2 - 6$

| x | y |
|---|---|
| -3 | 3 |
| -2 | -2 |
| -1 | -5 |
| 0 | -6 |
| 1 | -5 |
| 2 | -2 |
| 3 | 3 |

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$y = x^2 - 6$

## *How do the 2 graphs compare?*

**We can describe how the 2 graphs compare verbally....**

"Translation" or "slide" down 6 units

**And we can describe how the 2 graphs compare mathematically, using what we call "mapping notation"**

**$y = x^2$**

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

**$y = x^2 - 6$**

| x | y |
|---|---|
| -3 | 3 |
| -2 | -2 |
| -1 | -5 |
| 0 | -6 |
| 1 | -5 |
| 2 | -2 |
| 3 | 3 |

Notice:
Each y-value had 6 subtracted from it.

**(x, y)** ⟶ **(x, y − 6)**

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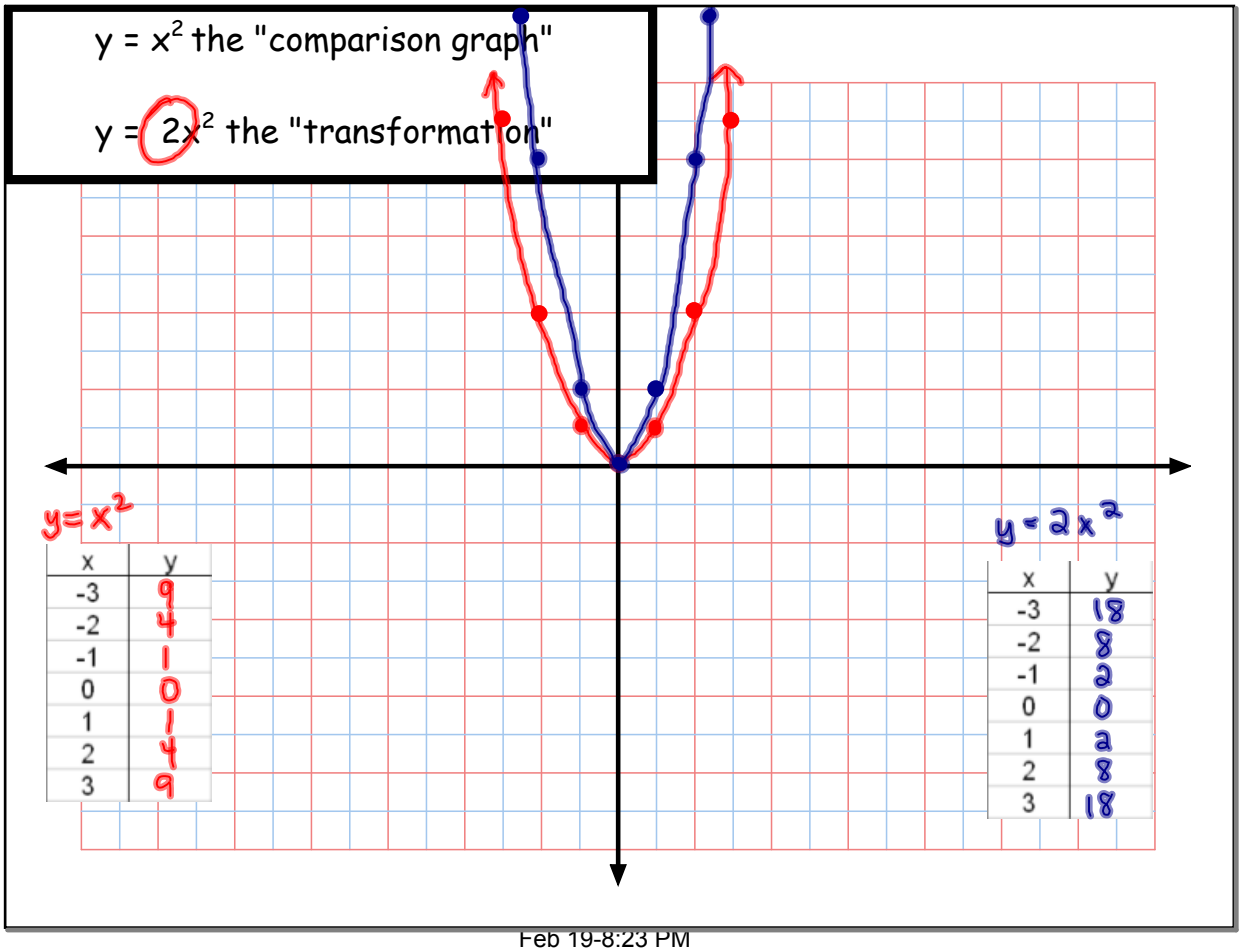
$y = x^2$ the "comparison graph"

$y = 2x^2$ the "transformation"



$y = x^2$

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

$y = 2x^2$

| x | y |
|---|---|
| -3 | 18 |
| -2 | 8 |
| -1 | 2 |
| 0 | 0 |
| 1 | 2 |
| 2 | 8 |
| 3 | 18 |

Feb 19-8:23 PM

$y = 2x^2$

## How do the 2 graphs compare?

**We can describe how the 2 graphs compare verbally....**

Vertical stretch of 2.
(2 times narrower)

**And we can describe how the 2 graphs compare mathematically, using what we call "mapping notation"**

**$y = x^2$**

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

**$y = 2x^2$**

| x | y |
|---|---|
| -3 | 18 |
| -2 | 8 |
| -1 | 2 |
| 0 | 0 |
| 1 | 2 |
| 2 | 8 |
| 3 | 18 |

Notice: Each y-value doubled.

**$(x, y) \longrightarrow (x, 2y)$**

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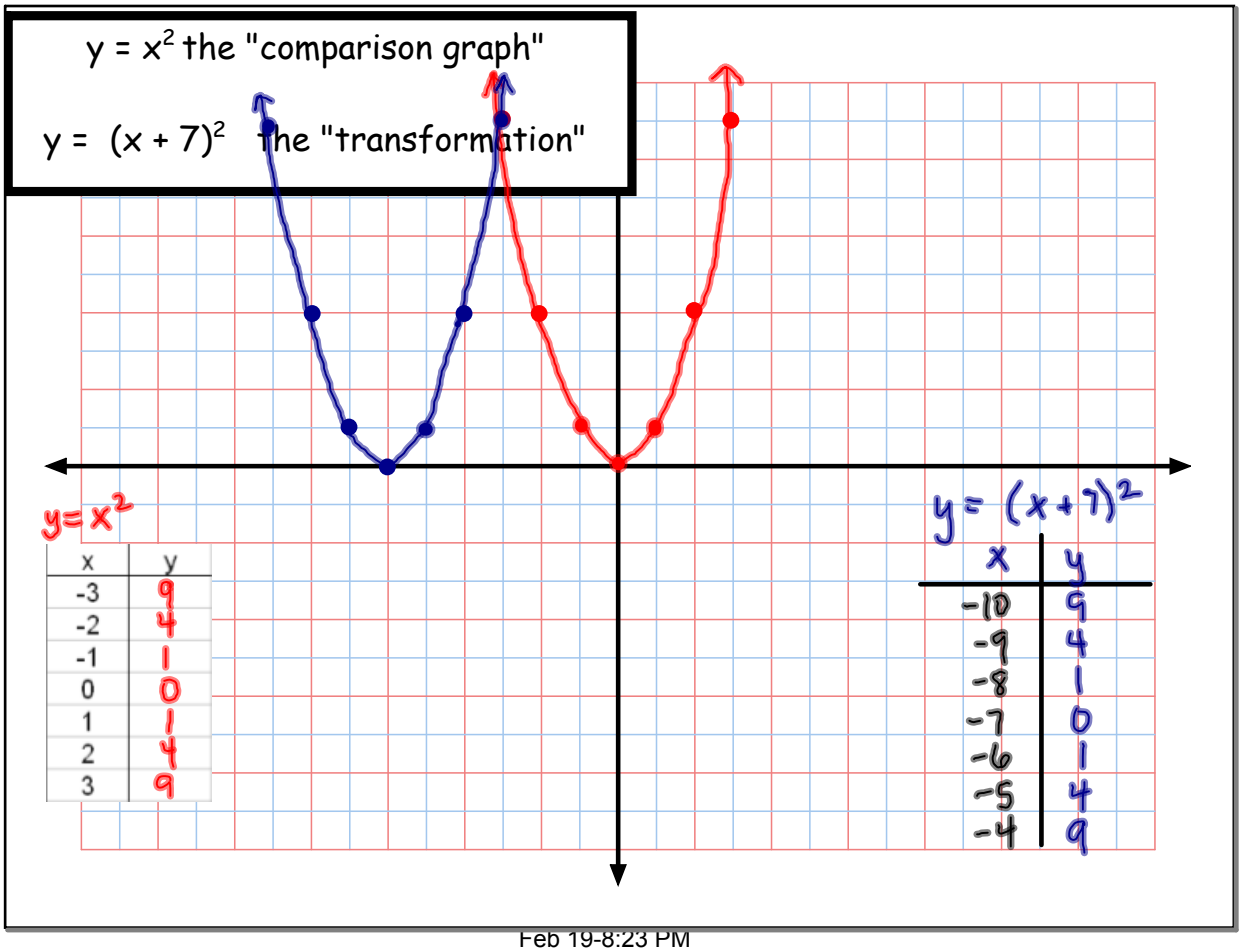
$y = x^2$ the "comparison graph"

$y = (x + 7)^2$ the "transformation"



$y = x^2$

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

$y = (x+7)^2$

| x | y |
|---|---|
| -10 | 9 |
| -9 | 4 |
| -8 | 1 |
| -7 | 0 |
| -6 | 1 |
| -5 | 4 |
| -4 | 9 |

Feb 19-8:23 PM

$y=(x+7)^2$

*How do the 2 graphs compare?*

**We can describe how the 2 graphs compare verbally....**

"Translation" or "slide" to the left 7 units

**And we can describe how the 2 graphs compare mathematically, using what we call "mapping notation"**

**$y = x^2$**

| x | y |
|---|---|
| -3 | 9 |
| -2 | 4 |
| -1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |

**$y = (x + 7)^2$**

| x | y |
|---|---|
| -10 | 9 |
| -9 | 4 |
| -8 | 1 |
| -7 | 0 |
| -6 | 1 |
| -5 | 4 |
| -4 | 9 |

**(x, y)** ⟶ $(x-7, y)$

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# Standard Form Vs. Transformational Form

The **standard form** of an equation is the form you are used to using

***- it has y by itself***

**$y = -x^2$** **$y = -x^2 - 1$** **$y = x^2 + 8$**

The **transformational form** of an equation is a form that has the $x^2$ by itself

**$y = -x^2$** **$y = -x^2 - 1$** **$y = x^2 + 8$**

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If you are given an equation that is not in standard form, you will need to rearrange it so that it is.

This is necessary to create a table of values!

Feb 10-3:50 PM

## Copy & Complete the following table:

SF - Standard form  Ref - Reflection  VT - Vertical Translation
TF - Transformational form  VS - Vertical Stretch  HT - Horizontal Translation.

| SF<br>y= | TF<br>$=x^2$ | Ref<br>(yes/no) | VS<br>y/n | VT<br>up/down | HT<br>left/right | Mapping Notation<br>(x,y)→( , ) |
|---|---|---|---|---|---|---|
| $y=x^2+1$ | | | | | | |
| $y=x^2+4$ | | | | | | |
| $y=x^2-2$ | | | | | | |
| $y=-x^2+1$ | | | | | | |
| $y=-x^2+7$ | | | | | | |
| $y=x^2-6$ | | | | | | |

Feb 10-9:47 PM

## Copy & Complete the following table:

SF - Standard form  Ref - Reflection  VT - Vertical Translation
TF - Transformational form  VS - Vertical Stretch  HT - Horizontal Translation.

| SF<br>y= | TF<br>$=x^2$ | Ref<br>(yes/no) | VS<br>y/n | VT<br>up/down | HT<br>left/right | Mapping Notation<br>(x,y)→( , ) |
|---|---|---|---|---|---|---|
| $y=x^2+1$ | $y-1=x^2$ | no | no | up 1 | no | $(x,y)\to(x,y+1)$ |
| $y=x^2+4$ | $y-4=x^2$ | no | no | up 4 | no | $(x,y)\to(x,y+4)$ |
| $y=x^2-2$ | $y+2=x^2$ | no | no | down 2 | no | $(x,y)\to(x,y-2)$ |
| $y=-x^2+1$ | $-(y-1)=x^2$ | yes | no | up 1 | no | $(x,y)\to(x,-y+1)$ |
| $y=-x^2+7$ | $-(y-7)=x^2$ | yes | no | up 7 | no | $(x,y)\to(x,-y+7)$ |
| $y=x^2-6$ | $y+6=x^2$ | no | no | down 6 | no | $(x,y)\to(x,$ |

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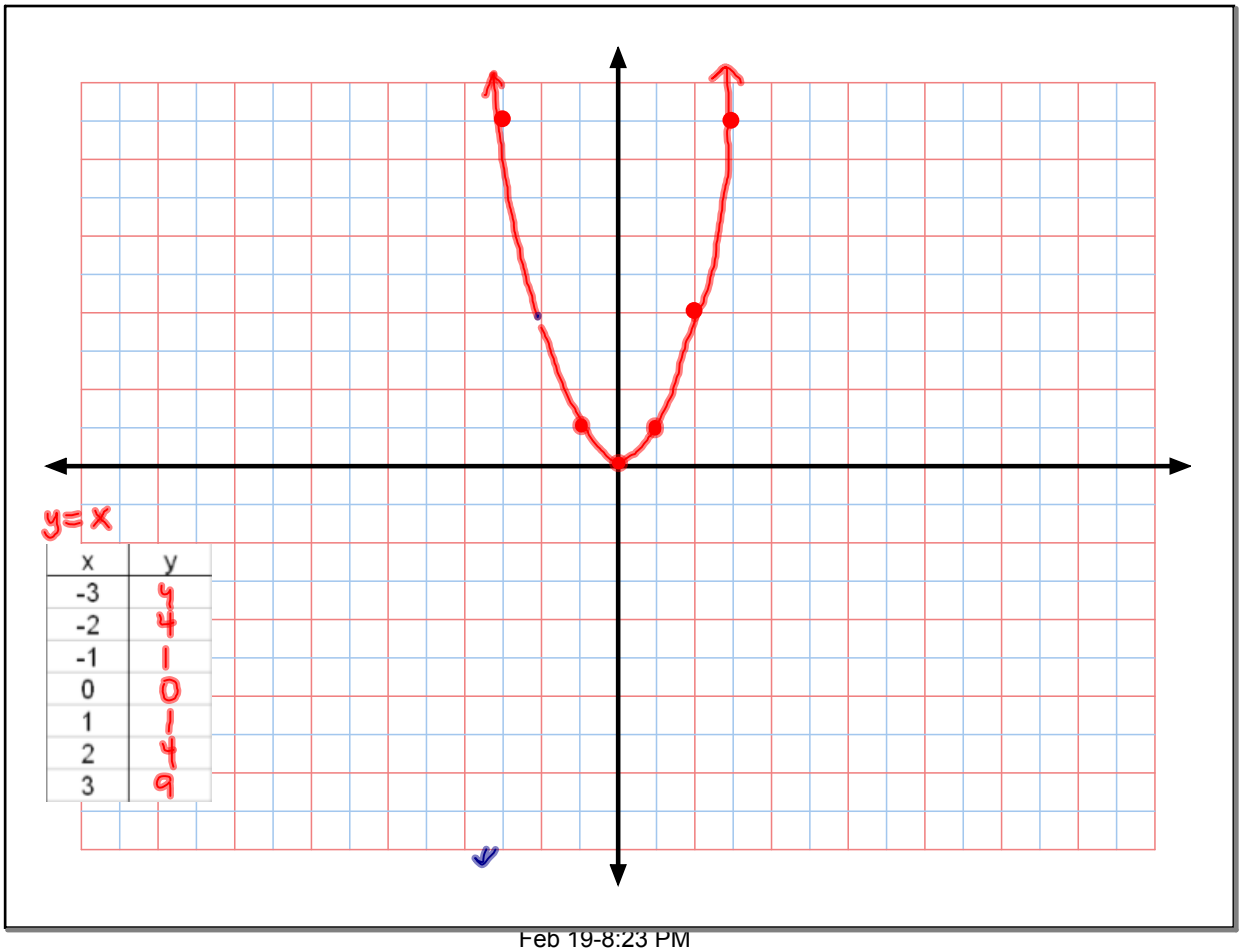


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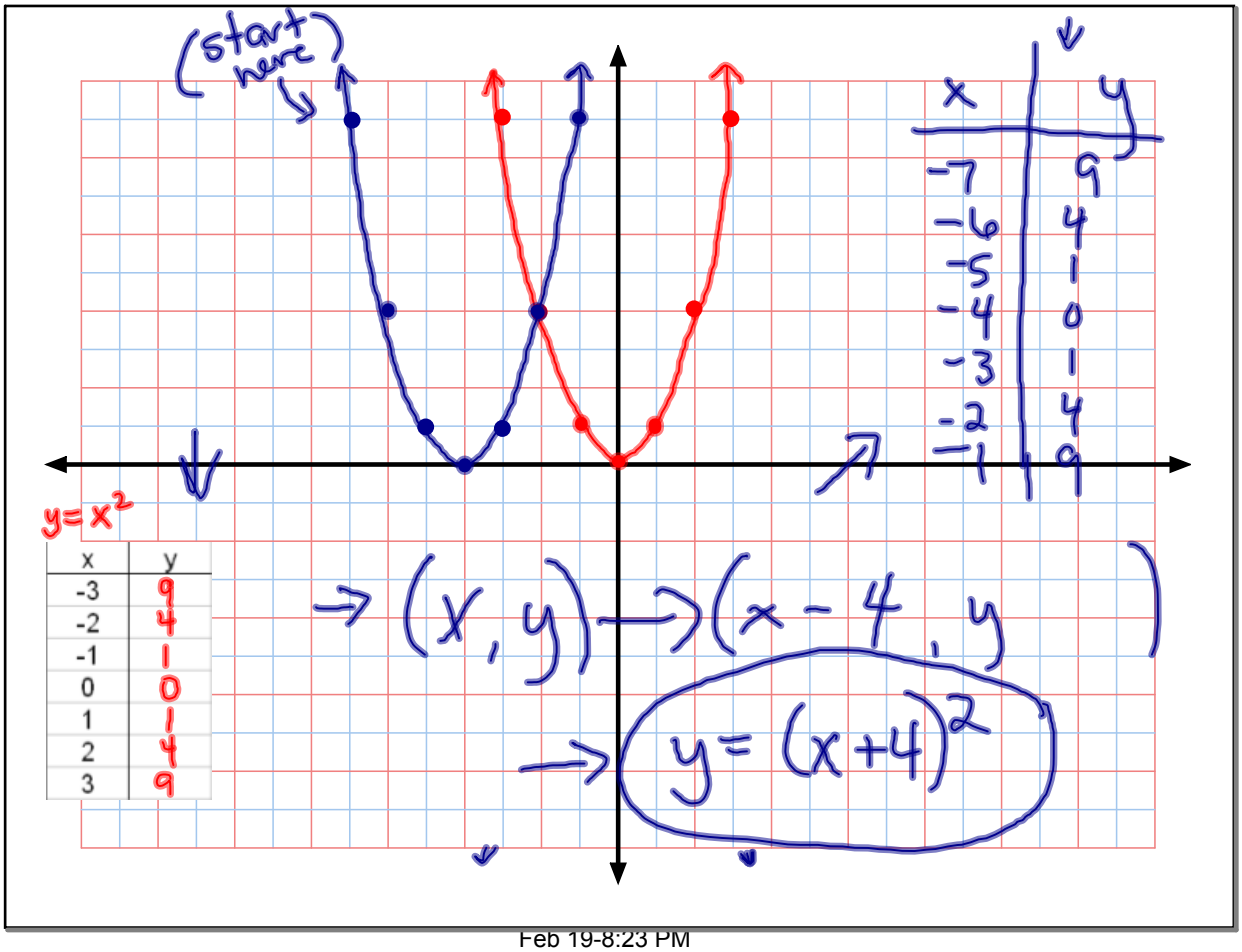


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## Copy & Complete the following table:

| SF | Ref (yes/no) | VS | VT | HT | Mapping Notation |
|---|---|---|---|---|---|
| $y=(x+3)^2$ | | | | | |
| $y=(x-2)^2$ | | | | | |
| $y=(x-5)^2$ | | | | | |
| $y=-(x+6)^2$ | | | | | |
| $y=-(x-4)^2$ | | | | | |
| $y=-(x-5)^2$ | | | | | |

$y=(x+3)(x+3)$ FOIL

$y=x^2+3x+3x+9$

$y=x^2+6x+9$

$-6x \quad -9$

$y-6x-9=x^2$

Feb 10-9:47 PM

## Copy & Complete the following table:

| SF | TF | Ref (yes/no) | VS | VT | HT (left or right) | Mapping Notation |
|---|---|---|---|---|---|---|
| $y=(x+3)^2$ | $y-6x-9=x^2$ | no | no | no | Left 3 | $(x,y)\to(x-3,y)$ |
| $y=(x-2)^2$ | | no | | | Right 2 | $(x,y)\to(x+2,y)$ |
| $y=(x-5)^2$ | | no | | | Right 5 | $(x,y)\to(x+5,y)$ |
| $y=-(x+6)^2$ | | yes | | | Left 6 | $(x,y)\to(x-6,-y)$ |
| $y=-(x-4)^2$ | | yes | | | Right 4 | $(x,y)\to(x+4,-y)$ |
| $y=-(x-5)^2$ | | yes | | | Right 5 | $(x,y)\to(x+5,-y)$ |

$y=(x+3)^2$

$y=(x+3)(x+3)$ FOIL

$y=x^2+3x+3x+9$

$y=x^2+6x+9$

$-6x\ -9$

$y-6x-9=x^2$

Feb 10-9:47 PM

## Complete the worksheet on Vertical Translations

Feb 10-4:04 PM

Feb 16-9:59 AM

## Warm Up

Feb. 14

1. Write a description (Reflection, Vertical Stretch, Vertical or horizontal transformation?) and the mapping notation of the transformation of the following:

a. $y = -x^2$

b. $y = x^2 + 6$

c. $y = -2x^2 - 3$

2. What is the difference between standard and transformational form?

Feb 13-9:59 AM

# Investigation Review Assignment

Feb 10-4:00 PM

## Section 4.3 Homework Questions

Pg. 175-184
#5,7,9,10,11,15,17,20,21,22

#5. b, c, f

#7 a) $y=\frac{1}{3}x^2$ b) $y=\frac{1}{5}x^2$ c) $y=3x^2$

widest?

Pg. 177

#9 a) $2\frac{1}{2}y=x^2$ ~~b) $y=\frac{1}{4}x$~~

~~a) $\frac{5}{2}y=\frac{1}{5}x^2$~~ ~~c) $y=\frac{1}{10}x^2$~~

$y=\frac{2}{5}x^2$ d) $y=x^2$

widest? →

c b a d

iii iv i ii

ii

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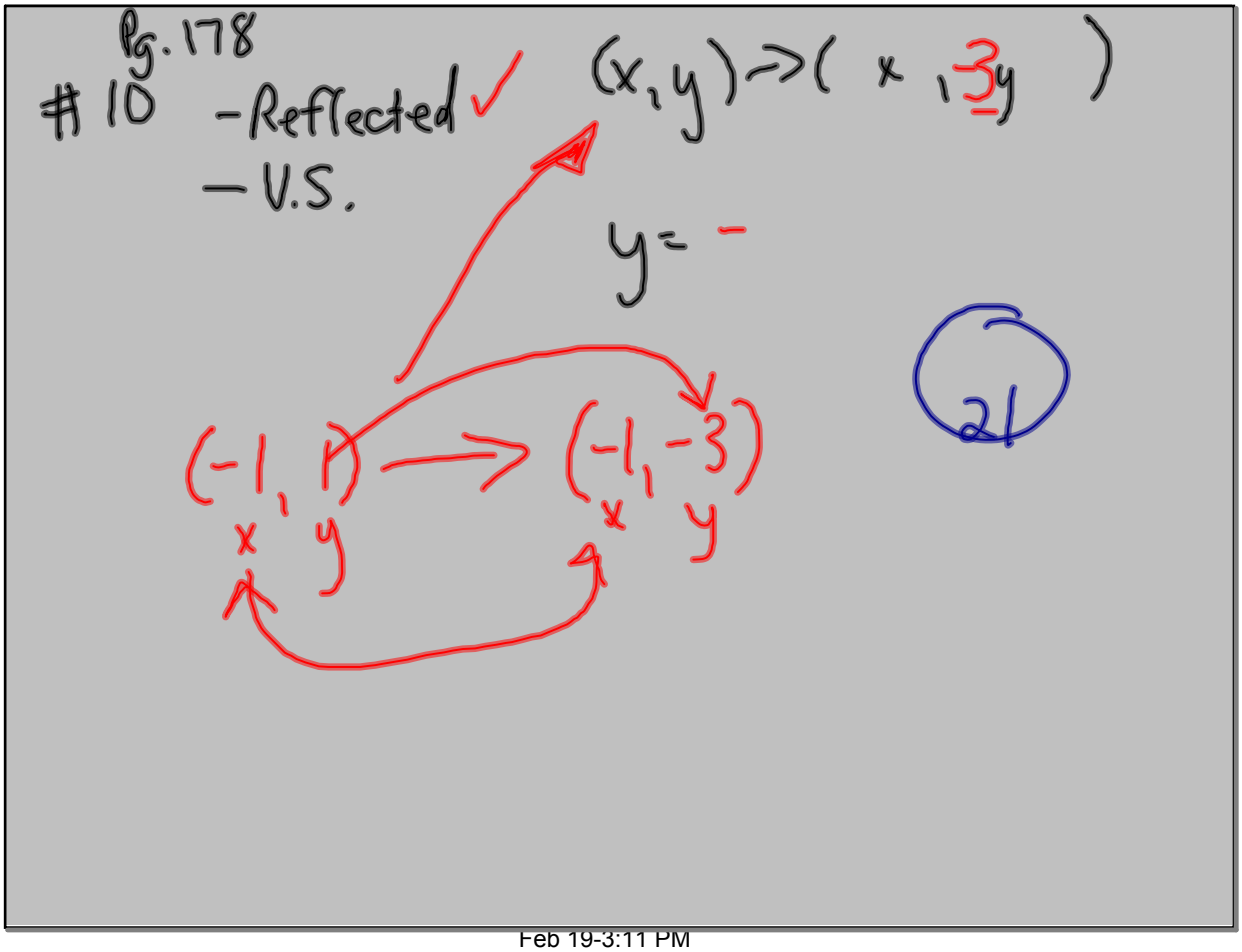


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**Notes to help you remember the types of transformations:**

1. **Reflection**
- this means that the graph flips
- the equation: $y = -x^2$
- the mapping rule: (x, y) -> (x , -y)

2. **Vertical Stretch**
- this means that the graph gets skinnier or wider
- the larger the V.S. value, the skinnier the graph
- the equation: $y = 2x^2$
- the mapping rule: (x, y) -> (x, 2y)
- this graph gets skinnier by a V.S. of 2

Feb 19-8:34 AM

3. Vertical Translation

- this means that the graph moves up or down
- the equation: $y = x^2 + 5$
- the mapping rule: $(x, y) \rightarrow (x, y + 5)$
- this graph moves up 5

4. Horizontal Translation *

- this means that the graph moves left or right
- the equation: $y = (x - 3)^2$
- the mapping rule: $(x, y) \rightarrow (x + 3, y)$
- this graph moves right 3 (always the opposite of what is in the equation)

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These are the ONLY transformations that you will see. The numbers for each transformation are ALWAYS located in the same place in the equation and in the mapping rule.

In order to determine and describe the type of transformation, you will need to rearrange the equation into standard form.

Feb 19-3:28 PM

## Other Terms:

### Quadratic Function

- $y = x^2$ is a quadratic function
- when graphed, a quadratic function makes a U-shape.

### Transformation

- when a graph changes from it's original shape (Ref.,V.S.,V.T.,H.T.)

### Mapping Rule

- A notation that describes how a graph and its image are related.

Feb 19-2:02 PM

**Math 10 Section 4.3 Practice Quiz** **Name:**_____

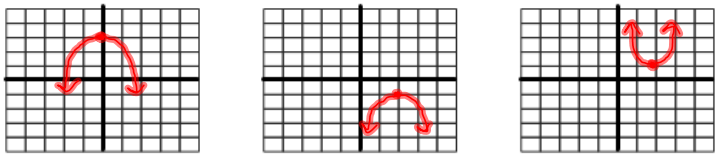
1. Fill out the following table, compare each to $y = x^2$

| Equation Given | Standard Form | Description in words | Mapping Notation | Sketch of graph |
|---|---|---|---|---|
| $2(y + 1) = x^2$ | | | | |
| $-y = (x - 4)^2$ | | | | |

2. What are the four types of transformations? Define each. Use diagrams if necessary.

3. Which of the following would have the widest graph?
   a. $3y = x^2$ b. $\frac{1}{4}y = x^2$ c. $6y = x^2$

4. Match the following equations with the correct graph below.
   a. $y = (x - 2)^2 + 1$ b. $-(y + 1) = (x - 2)^2$ c. $-(y - 3) = x^2$



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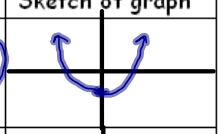
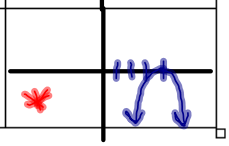
**Math 10 Section 4.3 Practice Quiz** Name: _____

1. Fill out the following table, compare each to $y = x^2$

| Equation Given | Standard Form | Description in words | Mapping Notation | Sketch of graph |
|---|---|---|---|---|
| $\frac{1}{2}(y+1) = \frac{1}{2}x^2$ | $y = \frac{1}{2}x^2 - 1$ | -V.S. is $\frac{1}{2}$<br>-moved down 1 | $(x,y) \to (x, \frac{1}{2}y - 1)$ |  |
| $-y = (x-4)^2$ | $y = -(x-4)^2$ | -Reflected<br>-moved right 4 | $(x,y) \to (x+4, -y)$ |  |

2. What are the four types of transformations? Define each. Use diagrams if necessary.

Reflection → flipped over x-axis
V.S. → wider or skinnier
V.T. → moves up or down
H.T. → moves left or right

3. Which of the following would have the widest graph?

a. $3y = x^2$ $y = \frac{1}{3}x^2$

b. $\frac{1}{4}y = x^2$ $y = 4x^2$

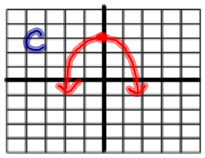
c. $6y = x^2$ $y = \frac{1}{6}x^2$

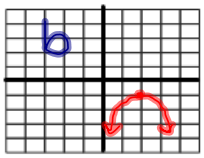
4. Match the following equations with the correct graph below.

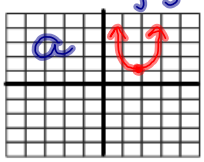
a. $y = (x-2)^2 + 1$

b. $-(y+1) = (x-2)^2$

c. $-(y-3) = x^2$ $y - 3 = -x^2$ $y = -x^2 + 3$

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